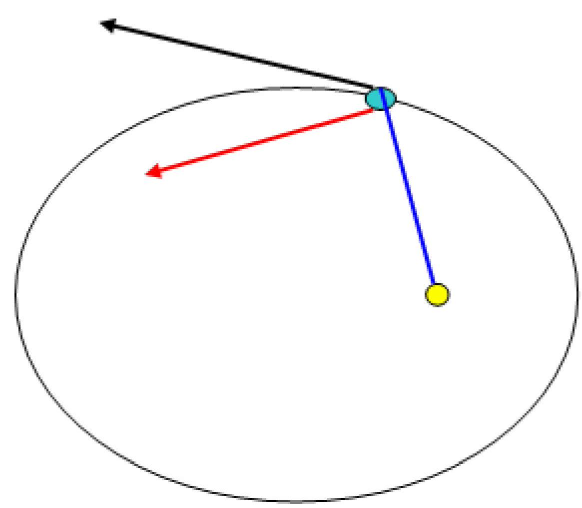
Orbits Explained

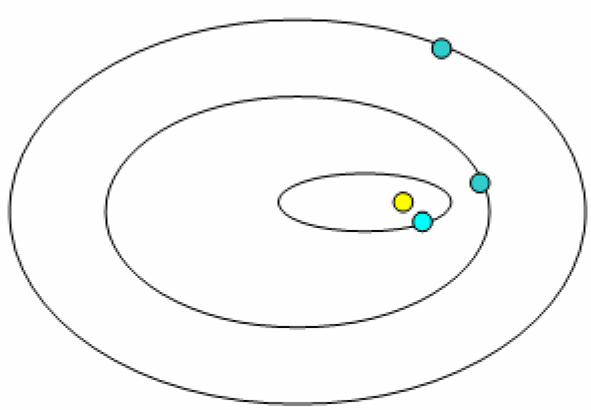
An Overview:

A Priori Proof for Kepler's Planetary Laws and the Energy Equation for Orbits

This is a synopsis of the a priori set of proofs presented in *Orbits Explained*. Those who are familiar with the concepts of vectors will be able to grasp this material easily. A complete a priori proof for Kepler's First Law, the Law of Ellipses, is presented in the first part of this overview. The scheme for the other proofs is given complete in terms of logic but incomplete in terms of mathematical detail. Otherwise the overview would be too cumbersome. The overview begins with some elementary vector instruction in order to give the pure beginner a fighting chance to follow this short and convenient synopsis of *Orbits Explained*. In other words, - "Welcome, everyone."

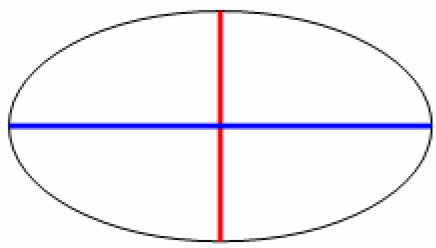


For a start, in the drawing above, a planet drawn in blue is orbiting the Sun. We will call the imaginary straight line to the Sun, the "distance radius" or at times simply the "radius". It is represented by the solid blue line. The actual direction and speed of travel are indicated by the black arrow. The arrows in the diagram are examples of velocity vectors. A velocity vector is an arrow whose length and orientation represent the speed and direction, respectively, of travel. The black arrow represents the total velocity of the planet and lies on a line that is the tangent to the orbit. The tangent is defined to be the line that grazes the curve of the orbit, touching it at only one point, without crossing it. The red arrow represents a different aspect of the velocity; it relates to the speed of the planet which is measured solely in the direction that is at a right angle to the distance radius to the Sun. This velocity is known as tangential velocity (do not be confused by terminology – the tangential velocity is not along the tangent line to the ellipse – the total velocity is). Tangential velocity and distance to the Sun are of prime concern to us throughout the a priori proof. As is true of all vectors, the length of an arrow can be adjusted so that it represents the velocity of the planet. For example, if the planet was at a position in the orbit where it traveled slower, the length of the arrow would decrease.



In the drawing above, there are several planets orbiting the Sun. Each orbit differs in size from the others. Each will be shown to require a different amount of time to complete one orbit around the Sun. The time it takes to orbit once is known as the period.

In the drawing below, the longest dimension of an orbit is the major axis. It is drawn in blue. Half the distance is referred to as the semimajor axis. The minor axis is drawn in red.



Kepler's Three Planetary Law's describe the behavior of planets. These Laws were based on Kepler's brilliant analysis of astronomical observations.

The primary aim of *Orbits Explained* is to explain Kepler's Three Planetary Laws and two additional phenomena without using any astronomical observations:

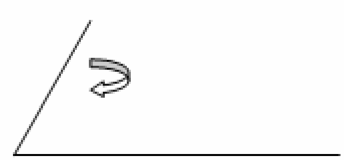
1) Planets travel in elliptical orbits. This is Kepler's First Planetary Law.
2) Planets sweep out equal areas in equal times. This is Kepler's Second Planetary Law. Newton explained this in *a priori* fashion.
3) Planets' periods are proportional to the square root of the cube of their semimajor axis.   This is Kepler's Third Planetary Law.

4) The gravitational force on a planet decreases with the square of the distance to the Sun. This is Newton's Inverse Square Law of Force.
5) Throughout its orbit a planet lacks a constant amount of energy that it would need to gain in order to escape from the Sun. I refer to this as the Law of Planetary Capture.

As proofs of these five laws are demonstrated, it will become evident that they are fundamentally interrelated.

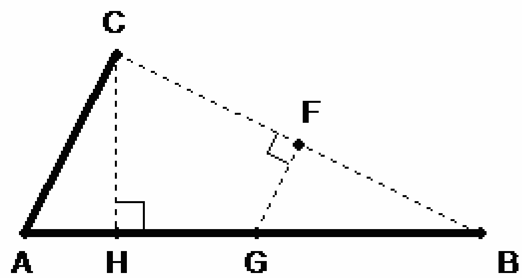
All of the five planetary laws mentioned above are explained by introducing an original mathematical device called the hododyne. Another name for the hododyne is the Inverse Proportion Machine. It will be described superficially in this overview of the a priori proofs. The formal geometric and algebraic proofs for the hododyne are found in Chapters 8 and 9 of the full text of *Orbits Explained*. The hododyne can be visualized as an immense template, larger than the orbit itself, engine-like, regulating the path and the speed of the planet. Specifically, the hododyne is responsible for compelling the planet to behave with a given speed and direction in accordance with what is proper for the planet's position with respect to the Sun.

Remarkably, the hododyne is simply a line that is bent so that the resulting two parts can be made to spin around each other:

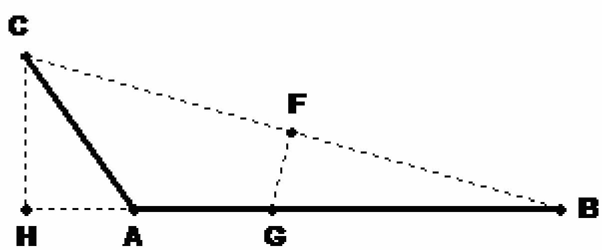


How can something so apparently simple explain something as complex as an orbit? The answer lies in the act of assigning landmarks to the parts of the hododyne.

One of the landmarks will represent the position of the Sun. Another will represent the position of the planet. Other landmarks will simply be geometrical locations on the spinning parts of the hododyne. Specifically, we create landmarks of the hododyne by first connecting the far ends of the bent parts by a straight line; then perpendicular lines are drawn at specified locations.



In the hododyne above, AC and AB are the two spinning parts of the hododyne. The point F bisects the connecting line CB. Segments FG and HC are created so that they are perpendicular to segments CB and AB respectively. When necessary, the segment AB is given an extension so that a perpendicular can be drawn to meet point C as has been done in the arrangement below:

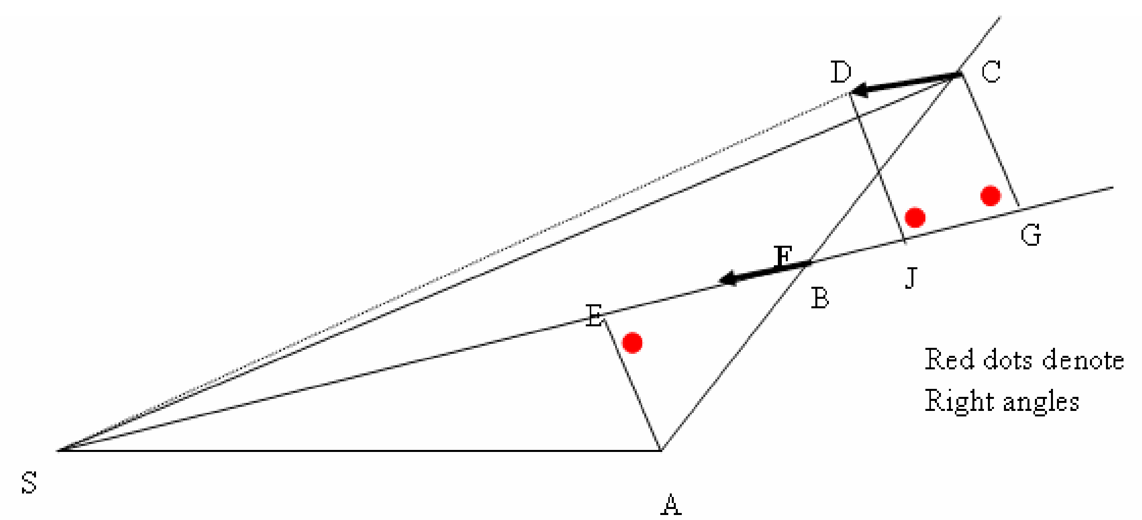


Chapter 9 of the full text of *Orbits Explained* contains the mathematical proof that for a hododyne as labeled above, segment AG is inversely proportional to segment HB. In other words as the two parts of the hododyne spin, the lengths of these segments change; one grows while the other diminishes in such a fashion that their numerical product is a constant; the length of HB times the length of AG is a constant.

The recognition of the intrinsic inverse proportion properties of the hododyne invites us to look for a behavioral match in nature which involves properties that are inversely proportional to each other. These suitable properties could be assigned to the segments of the hododyne that are inversely proportional to each other. Such a match does indeed exist for the planetary properties of distance to the Sun and tangential velocity. The inverse proportion between distance to the Sun and tangential velocity is the root of the entire set of *a priori* proofs presented in *Orbits Explained*. Let's see why tangential velocity and distance to the Sun are inversely proportional.

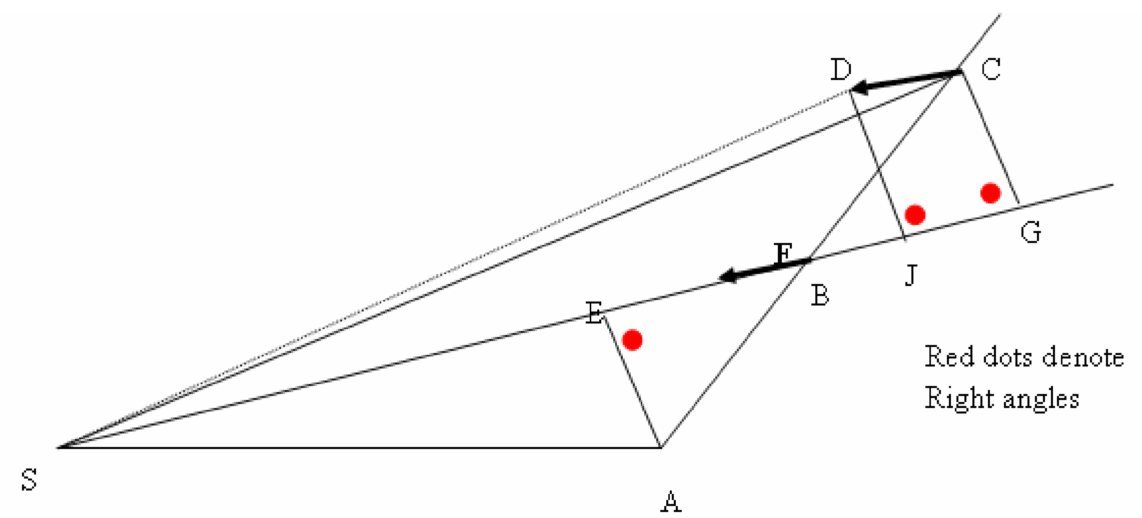
In the course of proving that a planet's radius to the Sun sweeps equal areas in equal times, Newton, in essence, simultaneously proved that tangential velocity is inversely proportional to distance to the Sun. So, let's examine Newton's *a priori* proof regarding equal areas swept in equal times. We will follow the general scheme of Newton's Proof as presented in *Feynman's Lost Lecture, The Motion of Planets Around the Sun,* by David Goodstein and Judith Goodstein, published by Norton Press.

Newton's diagram is reproduced below. In Chapter 5 of *Orbits Explained*, Newton's proof and diagram will be analyzed in detail. Briefly here, suppose that a body is in motion at a constant speed, not subject to gravity.



We are interested in the area swept by the line which connects the moving body to the point, S. Let the body move from A to B in a unit of time and then from B to C in an equal unit of time. Since its speed is constant, the distance AB will equal the distance BC. Now bear with a little geometry. Angle ABE is equal to angle CBG by the rules of intersecting lines. Angles AEB and BGC are right angles. Side AB is equal to side BC. Thus the right triangles AEB and BGC are equal since they are both have the same three angles and the same length of hypotenuse. So, side GC is equal to side AE. The area of a triangle is equal to the base times the height divided by two. The area of triangle SAB is equal to the area of triangle SBC since they share a base SB and their heights, AE and GC, are equal. So, in the absence of gravity, as the object moves in equal times from A to B and then from B to C, the corresponding areas swept, SAB and SBC, are equal. Thus equal areas are swept in equal times in the absence of gravitational force.

In Chapter 5 of *Orbits Explained* it will be shown, using similar triangles in Newton's fashion, that in a gravitational field, the planet moves from B to D instead of from B to C. It will also be shown that the area swept will be SBD and that this area is equal to the area swept in the absence of gravity, area SBC. In other words the gravitational force does not change the amount of area swept. And thus, even in the presence of gravity, equal areas are swept in equal times.

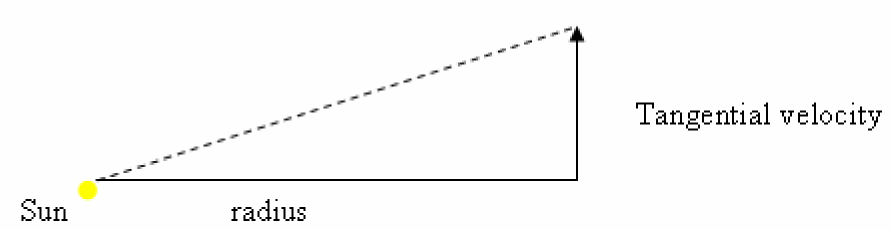


We therefore see that Newton showed the following two statements to be true: As an object moves past a stationary position at constant speed, it sweeps out equal areas in equal times. If the moving object is subject to a force attracting it to a stationary position, the object will still sweep out equal areas in equal times.

Now let's see how equal areas swept in equal times can be restated in terms of an inverse proportion so that we can apply inverse proportion properties to the hododyne's segments HB and AG. As stated a few paragraphs above, the inverse proportion inherent in the phenomenon of equal areas swept in equal times relates two things to each other:

1) The distance to the Sun
2) The speed of the planet as measured along a line that is perpendicular to the direction towards the Sun. This particular aspect of speed is conventionally labeled the tangential velocity.

In the diagram below it will be shown that for a given time, the triangle of area swept is measured by its base, the radius distance to the Sun, and its height which is determined by the tangential velocity.

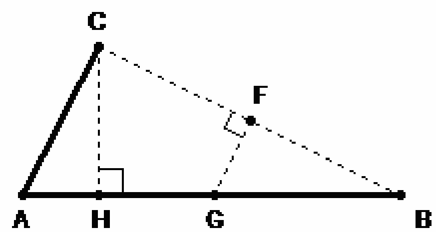


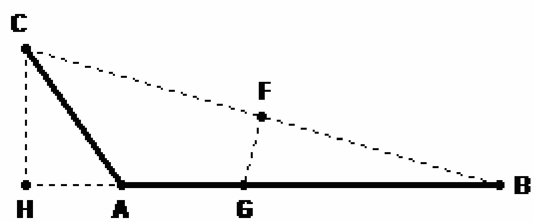
The area of a triangle is one half the base times the height. Bear in mind that we are considering what happens to areas swept during equal time intervals. Note that for any fixed time interval, the length of the distance traveled in the direction of tangential velocity is directly proportional to the tangential velocity itself since the product of time and velocity equals distance. This distance traveled becomes the height of the triangle of area swept. So for the triangle of area swept, in the diagram above, there are two important sides. One important side, the base, is the radius distance to the Sun. The other important side, the height of the triangle, essentially becomes the distance traveled in the fixed time interval in the direction of tangential velocity; this height is proportional to the tangential velocity for a fixed interval of time during an orbit. Thus to keep the area of the triangle constant, the tangential velocity and radius must be inversely proportional to each other. We have our two properties to assign to the hododyne.

Let's pause for a moment to analyze what has just been stated. A subtle and hidden logical choice was made. We know that equal areas are swept in equal times. For our a priori proofs we will examine the triangle of area swept for tiny instant of time. We logically choose to use tangential velocity instead of total velocity to give us the "height" of the triangle of area swept. The reason for this is that the total velocity can be broken into two components, one radial and one tangential. In an instant of time the radial velocity and motion does not sweep any area at all since it is directly along the line connecting the planet to the Sun. Radial and tangential velocity are presented formally in Chapter 17 of *Orbits Explained*.

We can return to our plan of exploiting the relationship between tangential velocity and distance to the Sun since we have a device, the hododyne, which generates segments that are inversely proportional to each other. All that needs to be done is to assign one segment to represent distance to the Sun and the other segment to represent tangential velocity. Then as the two parts of the hododyne are made to spin, the position and tangential velocity of the planet become evident, thus tracing the shape of the orbit as follows:

As mentioned above, as the parts of the hododyne spin, the angle between the parts might be less than or greater than 90 degrees. At present let's examine the two arrangements. As mentioned above, note two positions of the hododyne below to see that point H moves to an extension of line AB when the angle between the spinning segments AC and AB is greater than 90 degrees. These two representative arrangements of the parts of the hododyne will be incorporated into a single diagram farther below:

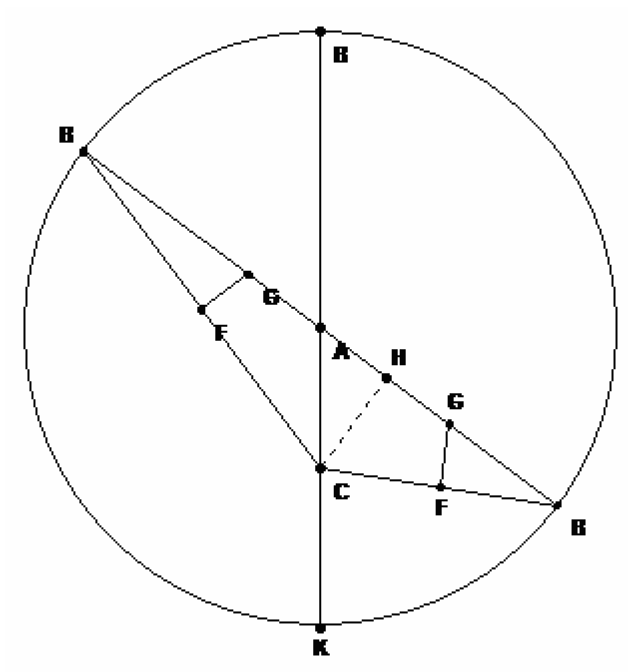




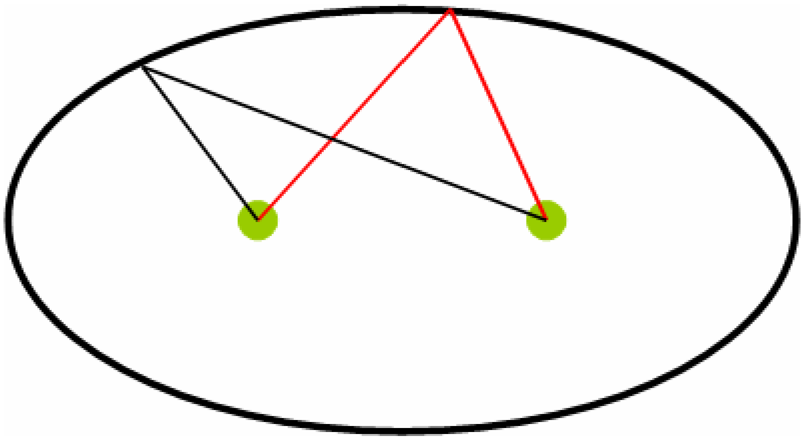
Note again that the two moveable parts of the hododyne are the segments AB and AC. The segment HB will be assigned to represent tangential velocity. The segment AG will be assigned to represent the radius distance to the Sun. These two assignments are consistent with the fact that, as stated above, these two segments of the hododyne are inversely proportional to each other; the radius distance to the Sun is inversely proportional to the tangential velocity.

Note in the diagram below that as the hododyne segment AB spins, the segment AB becomes the radius of a full circle that is traced by point B. The full circle that is traced is known as the hodograph. It was described by Sir William Rowan Hamilton in 1846. Hamilton based his hodograph on the empirical observations that orbits are elliptical and that force is inversely related to the square of the distance to the Sun. We will arrive at the hodograph proof via a priori methods.

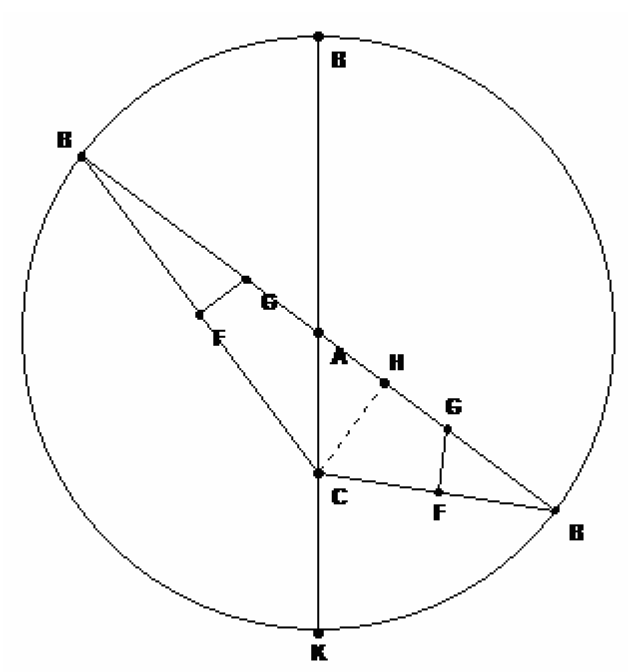
In anticipation of being able to determine the position and tangential velocity of the planet, we can draw two representative positions of the hododyne on such a hodograph circle. One hododyne position, drawn on the left side of the crcle, is arranged so that its segments AC and AB are less than 90 degrees from each other. The other hododyne position, on the right side of the circle, is arranged so that the angle between the segments is greater than 90 degrees:



Referring to the diagram above, we can see that the planet is always on an elliptical curve as follows:



By definition, a point is on an ellipse if its distances from two other fixed points sum to a constant. The two fixed points are called foci of the ellipse. The lengths of the red lines sum to the same length as the sum of the black lines in the ellipse above. Now back to the hodograph and hododyne:



Since for any hododyne, point F is a bisector of the connecting segment CB, there are two similar triangles created, CFG and BFG, when we draw the extra dotted line CG. G is the position of the planet. A is the position of the Sun at a focus of the ellipse. C is the other focus of an ellipse. It represents an empty point in space. How do we know that there is an ellipse and that G, the planet's position, is always on it and that A and C are foci? Look at the line AB. It is of constant length since it is the part of the hododyne that spins. Note that the length of CG added to the length AG must be equal to the length of AB since the length of CG is equal to the length of BG by similar triangles mentioned above. So BG + AG = BA = CG + GA. In other words CG = GA = a constant length equal to BA, the length of the spinning part of the hododyne. Now observe that C and A are fixed points since A is the bend between the parts of the hododyne and C is the end of one of the parts of the hod/dyne. By studying the diagram we can see that this in effect demonstrates that point G is on an ellipse since the sum of its distances to the fixed points A and C is a constant. Thus the planet at point G travels an elliptical path as indicated by the hododyne.

So, the hododyne has shown us in *a priori* fashion that planets travel in elliptical orbits.

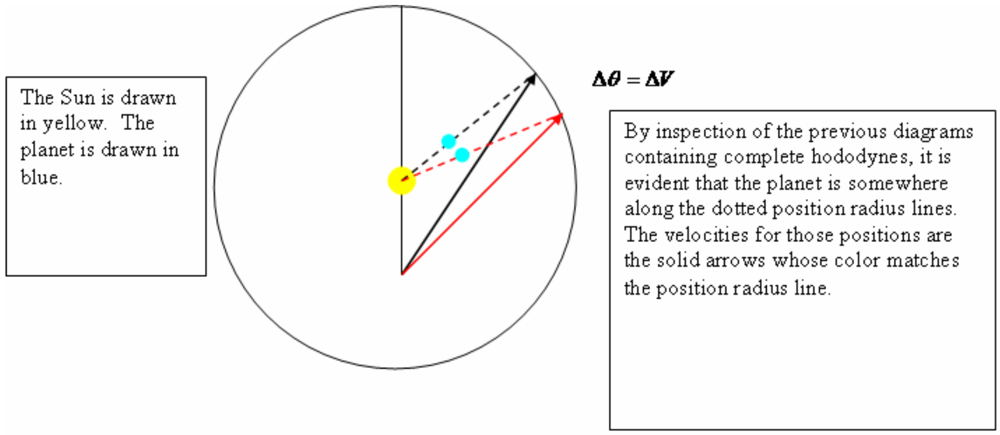
Using stepwise methods, it will be shown in the full text of *Orbits Explained* that as the segment AB spins around segment AC, the following planetary properties can be seen in the hododyne and hodograph:

1) As the hododyne part AB spins around the hododyne part AC, the length of the segment AG represents the distance from the planet to the Sun.
2) Point A represents the position of the Sun. Point G represents the position of the planet.
3) The direction of the segment AG indicates the direction of the line that connects the planet to the Sun.
4) When we interpret any of our hododyne generated diagrams, the velocities as drawn are ninety degrees away from their true directions. This ninety degree rotation is evident from inspection of the hododyne which represents the radius to the Sun, AG, and the tangential velocity, HB, on the same straight line. Since, by the definition of tangential velocity, this obviously can not be the case, we must always interpret the velocity direction to be ninety

degrees away from where it seems to be. This is the ninety degree correction rule for hodoynes. The true direction of tangential velocity is 90 degrees from segment HB.

5) By applying the rules for components of vectors and visualizing the path of the planet in relation to tangential velocity and the position of the Sun, the following properties of total velocity become evident: The direction of segment CB relates to the instantaneous direction of the planet's movement. But we must remember that the direction of movement is actually ninety degrees away from segment CB according to the ninety degree correction rule for hodoynes.
6) The length of the segment CB indicates how fast the planet is traveling.
7) In the course of its movement, the planet will have some of its motion occur perpendicularly to the line that joins the planet to the Sun. In other words, one component of the total velocity is the tangential velocity. The length of the segment HB indicates how fast the planet is traveling in this important direction. So, the length of the segment HB represents the tangential velocity. However, although the length of segment HB represents the magnitude of tangential velocity, the ninety degree correction rule for hodoynes gives the actual direction of tangential velocity.
8) Segment HC represents radial velocity. Radial velocity is the speed at which the planet travels along the line joining the planet to the Sun. Again, the ninety degree correction rule for velocity on the hodoyne applies.

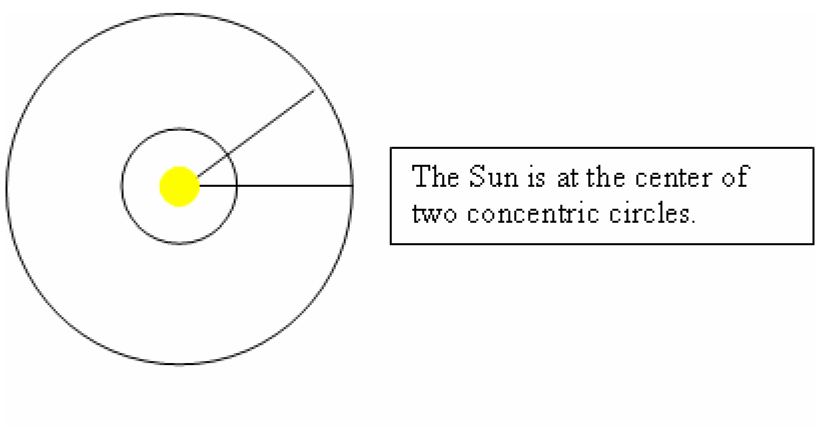
The hodoyne demonstrates how the velocity of the planet changes as the planet orbits the Sun. In the diagram below, we will see that the arc of the circle between the tip of the red arrow and the tip of the black arrow is representative of the angle, θ, through which the planet travels in relation to the Sun and is also representative of the change in velocity of the planet as it moves through that angle.



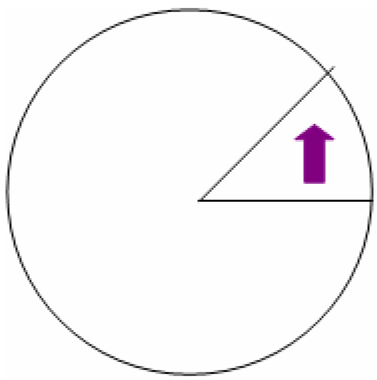
Momentarily, refer back to the diagram a few pages back with the two representative hododyne positions displayed and recall that it was stated that segment CB represents the speed and direction of the planet at any instant; from one orbital position to the next, the segment CB changes its size and orientation. Now in the figure above, CB is represented by the red solid line and then an instant later by the black solid line. This allows us to examine segment CB for two consecutive instants in time. Now, simply imagine a connecting line between the ends of the two consecutive segments. In accordance with the rules for addition of vectors, the connecting line represents the change in velocity. Note that all the CB segments start at point C. Their ends at point B trace out a circle as time passes and the planet moves. So ultimately the change in velocity, the connecting line for all the consecutive CB velocity segments, will be a circle for a trip once around orbit. Note that the arc of angle that the planet travels relative to the Sun can also be measured along the same circle since the direction of the planet relative to the Sun is in the direction of the segment CB. For every amount of arc of the circle traveled by the radius line connecting the planet to the Sun, there is therefore an equal amount of arc traced by the velocity segment CB. Thus the change in angle of position relative to the Sun is proportional to the change in velocity. Credit goes to Richard Feynman for finding this relationship between the angle swept and the change in velocity and to David Goodstein and Judith Goodstein for analyzing, clarifying, and publishing the derivation in the book cited above.

Our attention now turns to the a priori proof for the Inverse Square Law of Force and Distance. Bearing in mind that equal areas are swept in equal times, visualize the triangle of area swept in an instant. The planet in an elliptical orbit is at varying distances from the Sun. The wedge of area swept must be equal for equal times. The area of a wedge that is swept varies with the square of the radius distance to the Sun as follows:

In the two concentric circles below the wedge of area for each circle that is cut by the two lines must be proportional to the square of the radius of each circle since the entire area of each circle is proportional to the square of the radius of each circle.



The central angle in the two circles above is measured relative to the Sun at the center of each circle. We wish to examine the planet at different positions in its orbit but during equal time intervals. We know this means that equal areas must be swept. If the area swept must stay constant, and if we note that the two variables are the angle swept and the radius; it follows that if the area swept remains constant then the angle swept is inversely proportional to the radius. Stated otherwise, in order to keep the area of a swept wedge constant, if the angle decreases, the radius must increase in proportion to the square of the angle.   Furthermore, area is proportional to time since equal areas are swept in equal times. So it is true that for a given time interval, the change in angle must also be inversely proportional to the square of the radius. In any time interval the change in the angle can be interpreted to be synonymous with angular velocity. Angular velocity is the rate of change of the angle whose apex is at the Sun. It is the change in angle divided by the interval of time during which the angle changed. We call angular velocity W. So W is proportional inversely to the square of the radius. In the circle below the rate of change of the angle is determined by how fast the radius rotates. If W is small the radius rotates slowly.



The change in velocity per change in angle is constant per Richard Feynman as described above. This can be expressed as a fraction:

$\frac{\Delta V}{\Delta \theta} = constant$ where θ is the angle as measure from the Sun.

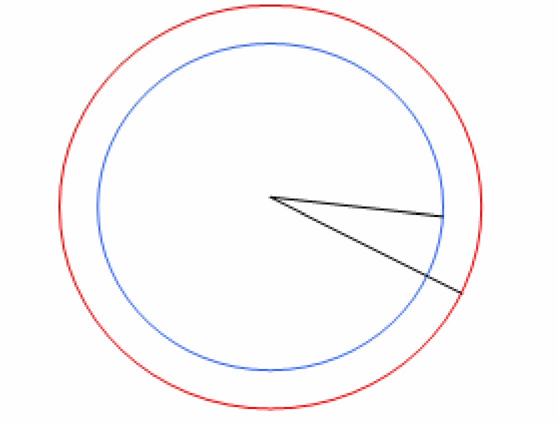
Force is proportional to $\frac{\Delta V}{\Delta T}$. By algebra it is valid to state the fraction differently as $\frac{\Delta V}{\Delta \theta} \times \frac{\Delta \theta}{\Delta T}$. Noting that $\frac{\Delta V}{\Delta \theta}$ is constant as described above, force $\frac{\Delta V}{\Delta T}$, is proportional to $\frac{\Delta \theta}{\Delta T}$. In the paragraphs above, W was defined to be the angular velocity and shown to be inversely proportional to the square of the radius. Since W is inversely related to the square of the radius, and since force is proportional to W, force must also be inversely proportional to the square of the radius to the Sun. This proves the Inverse Square Law of Force in *a priori* fashion.

The proportion above correlates with the full empirical equation for the Inverse Square Law of Force: $F = \frac{GMm}{R^2}$. It is sometimes convenient to use the empirical form of the a priori equations for the sake of clarity.

Note that the inverse proportion between force and the square of the distance is expressed in the fraction. For a circular orbit, $F = \frac{mV^2}{R}$ and the velocity can be expressed as $\frac{2\pi r}{T}$. Combining terms of the formulae for force and velocity and solving for the period we get $T = \sqrt{\frac{4\pi^2}{GM}} \times R^{\frac{3}{2}}$. So far we are considering only circular orbits.

From the formulae above for force and velocity it can be shown algebraically that velocity relates to radius by the standard formula for velocity in circular orbits: $V \propto \frac{1}{\sqrt{R}}$

However, look at what happens when circular orbits around the Sun are diagrammed and compared:

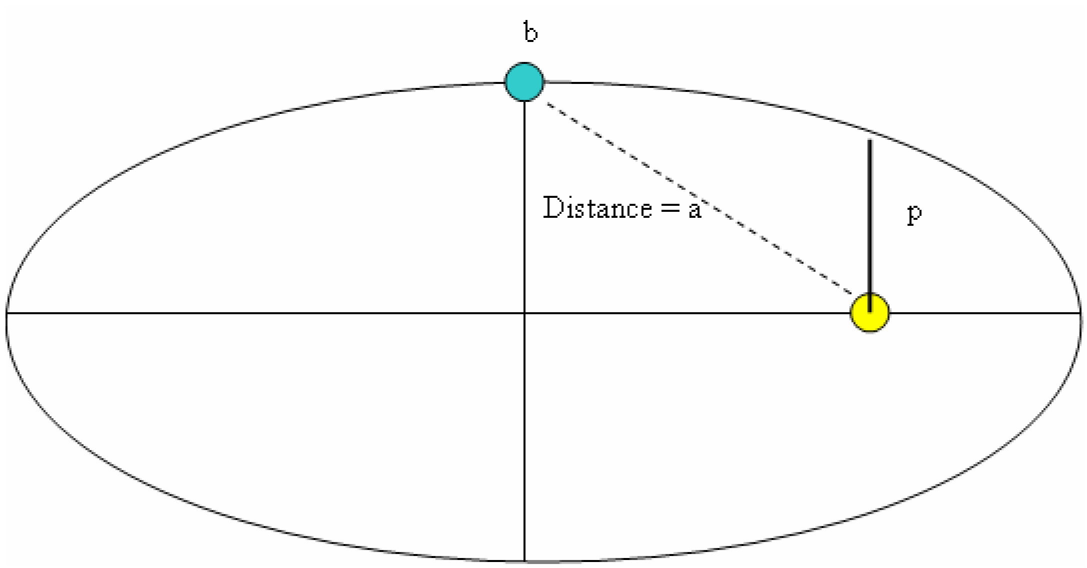


The radius is larger for the planet that is farther from the Sun. Imagine that the diagram is drawn on a grid such that some of the lines in the diagram represent distances whereas other lines on it represent velocities. The problem with the diagram is that the velocity as represented by a hododyne seems to become larger as the radius distance to the Sun also increases. But we know from the standard equation for velocity in circular orbits, $V \propto \frac{1}{\sqrt{R}}$, that the velocity must decrease. In order to correct the situation, the velocity must be scaled so that the velocity units of the diagram are proportional to $\frac{1}{R^{\frac{3}{2}}}$.

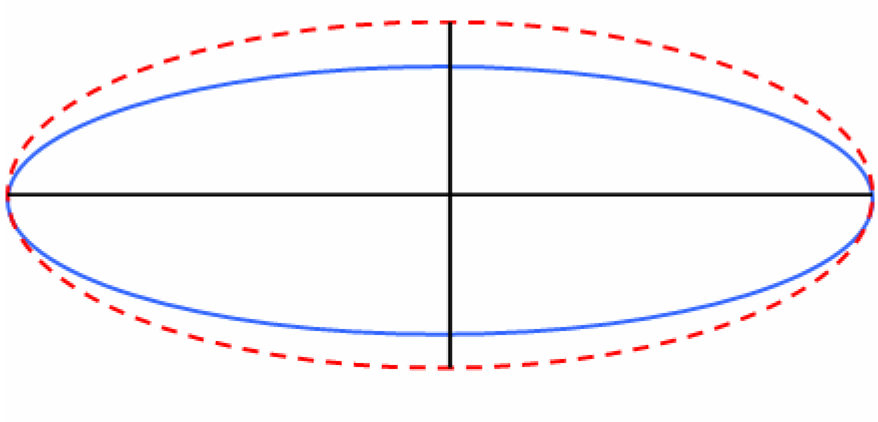
So we learn from studying circular orbits that a scaling scheme is necessary when analyzing hododynes and their resultant hodographs. Scaling schemes are an essential technique.

In the diagram below, the planet is at position b in the orbit. We will say that the planet is at position b when it is at the end of a semiminor axis as drawn above. The semilatus rectum, p, is traditionally defined as the perpendicular distance from a focus to a point on the ellipse. So we will informally say that the planet is at position p when it is at the tip of the semilatus rectum on the ellipse.

Position b is a special place since the planet is at a distance from the Sun that is equal to the length of the semimajor axis of the orbit. So pause for a moment and realize that two planets can orbit the Sun and have the same length semimajor axis but differ in semiminor axis length. The key point here is that the distance from the Sun when either planet is at its position b will be the same for both planets; the semimajor axis length, which is equal for the two planets. Note the distance equal to a, the semimajor axis length, from the planet to the Sun in the diagram below.



In the diagram below, the blue and red orbits have the same length semimajor axis but the red orbit has a longer semiminor axis.



The experience gained from comparing different size circular orbits cues us to obtain a proper scaling technique for interpreting the hododynes for elliptical orbits. The technique is found by studying the planet when it is at position b.

If we were to draw the hodograph for two different planets whose orbits have the same semimajor axis but different minor axis lengths on the same diagram, we would see that the diagram misleads us by portraying the tangential velocity of these planets as being proportional to the length of p, the semilatus rectum. But analyzing force reveals to us that this relationship can not be correct. Hence, since the distance to the Sun is the same for planets at b when the semimajor axis is the same, the force must be the same there as well according to the Inverse Square Law of Force. Accordingly, when actually diagramming these planets' hodographs, and when viewing the velocities and the way that they change in an instant of time, it becomes evident that for a planet at position b the velocity of the hodograph needs to be adjusted, or scaled, in order for force to be correct. Bearing in mind the law of Force during the inspection of a diagram with these two planetary hodographs, it becomes evident that the velocity for elliptical orbits must be scaled on the hodograph diagram by a factor of $\frac{1}{b}$ for orbits of equal major axis length and more generally by a factor of $\frac{1}{a}\times\frac{1}{\sqrt{p}}$ for orbits of unequal minor axis length where a is the semimajor axis length and p is the length of the semilatus rectum. In other words we can draw two different orbits of unequal major axis and unequal eccentricity on the same diagram. Just as we did for circular orbits we can apply our scaling method to learn the true behavior of the planet from our diagrams. The diagrams and analysis are straightforward but lengthy. Chapters 28 to 34 of *Orbits Explained* contain the full analysis.

In Chapter 33 of *Orbits Explained* it is shown that in the setting of analyzing orbits of equal major axis and unequal minor axis lengths, the scaling technique using the scaling proportion $\frac{1}{b}$ leads to the *a priori* proof of Kepler's Third Law which states that the periods of elliptical orbits obey the relationship $T \propto a^{\frac{3}{2}}$.

The procedure establishes the fact that elliptical orbits of equal semimajor axis length all have the same period as the circular orbit of radius equal to their semimajor axis. This allows the proportion relating the periods of circular orbits (as shown in Chapter 24 of *Orbits Explained*), $T \propto R^{\frac{3}{2}}$,to be generalized to apply to the periods of elliptical orbits $T \propto a^{\frac{3}{2}}$.

Orbital eccentricity can be defined mathematically. For our purposes here, we can say that long thin orbits are not at all circular in appearance; rather, we say that they are highly eccentric. The concept of escape velocity follows from both the concept of eccentricity and from the above scaling technique. A novel concept, the extremely eccentric ellipse gives insight into what would be necessary for a planet to escape its orbit. We make the orbit so incredibly long and thin that with one additional tiny thrust of force it will escape. In other words we look at the limit for how long and thin the orbit can possibly be and then exceed the limit so as to let the planet fly away. It is shown in Chapter 35 of *Orbits Explained* that escape velocity obeys the formula $V_{escape} = \sqrt{\frac{2GM}{R}}$ where G is the gravitational constant and M is the mass of the Sun.

In Chapters 37 to 39 of *Orbits Explained* it is shown that the hodograph can be analyzed and scaled so that a formula describing total velocity evolves: $V_{total}{}^2 = GM\left(\frac{2}{R} - \frac{1}{a}\right)$ where *R* is the current distance to the Sun and *a* is the length of the semimajor axis.

The similar terms in the formulae above for total velocity and escape velocity invite us to combine some of the terms in order to see if total velocity and escape velocity are somehow related during an orbit. Indeed the equation $V_{escape}{}^2 - V_{total}{}^2 = \frac{GM}{a}$ logically results. This is the Energy Equation for orbits. It tells us that for any position along an elliptical orbit, if the amount of energy equal to $\frac{GM}{a}$ is added, the planet will escape. This is the Law of Planetary Capture.

We will also see that the square of the escape velocity is twice the square of the velocity for a hypothetical planet in a circular orbit at the same specified distance from the Sun. In general, the square of velocity is related to the energy of motion for a planet. Thus, looked at another way, the Planetary Capture Law states that for any given position of a planet, twice the energy of a hypothetical circular orbit minus the actual energy due to total velocity will equal a constant everywhere along the orbit.

Notice that terms in the Energy Equation for total velocity squared, as written above, invite us to predict what will happen to the semimajor axis length of the orbit if the velocity is suddenly altered. A change in the semimajor axis will of course induce a change in the period of the orbit. Several examples of induced orbital changes will be demonstrated toward the end of *Orbits Explained.*

Along the way, several behaviors of planets become evident. One of them adds fundamentally to the understanding of the answer to the philosophical question; Why do planets stay in orbit? We can study the movement of a planet along its connecting line to the Sun. At some positions the planet may be moving farther from the Sun whereas at others it may be getting closer to the Sun. The velocity along this connecting line to the Sun is the radial velocity. If we analyze the hododyne in the diagrams of elliptical orbits we can see that there are pairs of radial velocities at 180 degrees from each other, or in other words, in opposite directions from each other. The hododyne shows that the radial velocity is always equal in magnitude but in opposite direction for positions that are on directly opposite sides of the Sun. So the planet never moves farther from or closer to the Sun during any pair of measurements at 180 degrees from each other. Since the orbit is simply a summation of all these pairs of movements, the sum of infinite pairs with zero net movement along the radius direction results in zero net movement towards or away from the Sun. And so the planet stays in orbit.

And so it is time for me to invite you to explore the main text of *Orbits Explained* to find justification for the findings outlined in this overview.

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