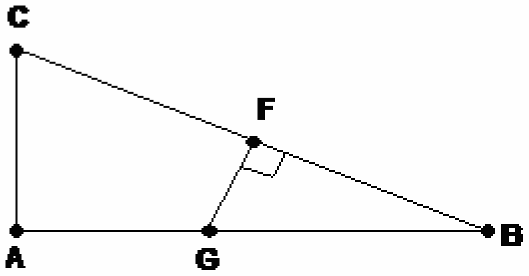
Any Right Triangle

In Chapter 6 we saw that there is an inverse relationship between distance to the Sun and the planet’s tangential velocity. This inverse relationship begs for a mathematical representation. In later chapters this mathematical representation will be shown to reside in the Inverse Proportion Machine. This chapter is a preliminary step in the derivation of the hododyne, the Inverse Proportion Machine.

Sometimes a small finding turns out to be useful. Here I present a finding regarding right triangles. I am not sure if it is unique or original and it surely seems innocuous enough to be unimportant. But it turns out to be the first step in the derivation of the hododyne, and so it is given here.

My purpose in this short chapter is to prove a property that holds true for all right triangles. In the next chapter concerning the Inverse Proportion Machine, we will be able to use this property as a mathematical tool.

Find two equal angles



In this right triangle, *BAC*, we have drawn $\overline{FG}$, a perpendicular to the midpoint of the hypotenuse, and let it intersect the base at point *G*. Now we have two right triangles *BAC* and *GFB* that are similar since they both share the angle *CBA*. (Since both triangles have a right angle and share angle *CBA* their remaining angles must also be equal since the sum of angles in a triangle must always equal 180 degrees. So angle *ACB* equals angle *FGB*.)

Examine sines of the equal angles

Next we will exploit the ratios of some of the sides of our triangles. We do this by examining the sine of the equal angles ACB and FGB. The sine of angle ACB must equal the sine of angle FGB since the angles are equal. Recall that the sine of an angle is a simple ratio. The sine of an angle is defined in the context of a right triangle that contains the angle. The sine of an angle is defined as the length of the side that is opposite the angle divided by the length of the hypotenuse of the right triangle. The sine is simply a ratio of lengths. The sine of a given angle is constant regardless of the size of the right triangle that contains it. In other words, imagine that as a specific right triangle grows in size, without changing its shape, from the size of a pea to the size of a house, the ratio of its sides does not change since the angles are not changed. Therefore we can safely say that equal angles have equal sines.

So, using the sines of the equal angles ACB and FGB, we see that $\frac{\overline{AB}}{\overline{CB}} = \frac{\overline{FB}}{\overline{GB}}$. Now since F is the midpoint of $\overline{CB}$, then $\overline{FB} = \frac{\overline{CB}}{2}$ so by substitution $\frac{\overline{AB}}{\overline{CB}} = \frac{\overline{FB}}{\overline{GB}} = \frac{\overline{CB}}{2\overline{GB}}$. Lastly, taking the first and last items in the equation, we see that $\frac{\overline{AB}}{\overline{CB}} = \frac{\overline{CB}}{2\overline{GB}}$. Cross multiplying to rid the fractions we see that $2\overline{GB}\left(\overline{AB}\right) = \left(\overline{CB}\right)^2$.

The Smaller Hypotenuse

This last equation reveals a property of our triangle that we can put to good use. Before stating the property in words, it will be convenient to give a general name to the segment that is represented in our figure by $\overline{GB}$. We will name it the *smaller hypotenuse.* We can see that a *smaller hypotenuse* results when the perpendicular bisector of the hypotenuse of any original right triangle intersects the original base, cutting the base into two new segments so that one segment becomes the hypotenuse of a new smaller right triangle. Having for convenience defined the *smaller hypotenuse,* we can state the property that is revealed in our last equation:

For any right triangle whose hypotenuse is bisected perpendicularly, the *smaller hypotenuse* times the original base that contains it, is equal to half the original hypotenuse squared.

We will call this relationship the “Smaller Hypotenuse Property” and will use it to derive the proof of the Inverse Proportion Machine.