

Tangential Velocity Area

We are still exploring the area swept by a planet as it travels along in its orbit. The sweep of area is due to that aspect of motion that is perpendicular to the radius line to the Sun.

We will examine the effects of distance from the Sun and tangential velocity on the area swept; it will be obvious that since equal areas must be swept in equal times, tangential velocity must be inversely related to the distance to the Sun. Our starting point will be Newton's *a priori* demonstration that equal areas are indeed swept in equal times, as was diagrammed in Chapter 5.

Where will this all lead? In Chapters 8 and 9 the Inverse Proportion Machine will be introduced; it will be shown to be a device that generates lines that are inversely proportional to each other in length. These lines of the Inverse Proportion Machine will be assigned to represent distance and tangential velocity in Chapter 11 so as to prove that orbits are elliptical.

So in this short chapter it is our easy task to understand the two properties that are inversely proportional to each other; distance to the Sun and tangential velocity.

The triangle of area swept

A triangle results from the arrangement of the distance from the Sun, which we will call the radius, and the tangential velocity, which is the velocity of the planet perpendicular to the radius. Simply let a small time occur and the tangential velocity results in a distance traveled in the direction of the tangential velocity. The result is a triangle representing the area swept.

The area of a triangle is one half times the base times the height. The area swept is thus proportional to the product of the tangential velocity, the height of the triangle, and the radius, the base of the triangle. Now since we showed in the previous chapter that equal areas are swept in equal times, the radius and tangential velocity must be inversely related since their product is always a constant. In the figure, the area of the triangle represents the area swept by the planet.

$$area = \frac{1}{2} radius \times V_t \quad \text{so :}$$

$V_t \propto \frac{1}{\text{radius}}$ if the area swept is constant for a constant

unit of time. And that is what we set out to demonstrate.

It is the central concept in the path to an *a priori* proof

that orbits are elliptical and force is inversely related

to the square of the distance from the Sun.

