Velocity Vectors

Sometimes language is a collection of words regulated by some form of grammar. In that case learning a foreign language is a matter of acquiring a vocabulary and receiving instructions about the grammatical rules. But learning the language of science is different. The building blocks of scientific ideas are not words. They are entire concepts. What kind of book is *Orbits Explained*? In a true sense this book is no different from a foreign language textbook. The scientific language consisting of concepts is first systematically presented, chapter by chapter. And then, when enough concepts are available the scientific discussion of orbits follows. In this chapter the concept to present is the vector.

In *Orbits Explained* we need to be able to describe motion graphically. We will use vectors to represent two different kinds of things. We will deal with both position vectors and velocity vectors.

Position vectors are essentially arrows that show the position of a planet in relationship to the position of the Sun. A position vector is literally like a tape measure that reaches from the Sun to the planet. The tape reaches a certain distance and in a certain direction.

Distance is certainly an easy concept. It is the fact that a direction is involved that differentiates a position vector from a simple numerical measurement of distance.

Velocity vectors are essentially arrows that describe what is happening to the planet at any given instant. Velocity vectors do not tell anything about the position of a planet. Instead, they are arrows that point in the direction in which the planet is moving; and the length of the velocity arrow represents the speed of the moving planet.

How to represent motion graphically

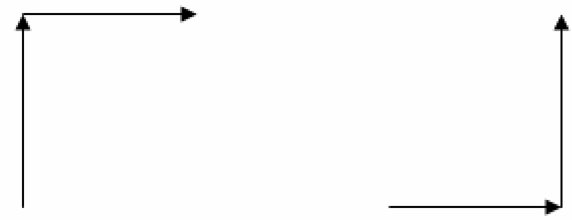
Any pilot or golfer is familiar with vectors even if they are not aware of them in a conscious sense. The flight path of the plane or the golf ball is determined by the thrust applied, but also by the wind speed and direction. Predicting the final result is a matter of simple vector analysis.

We use velocity vectors to represent the speed and direction of the planets. Vectors can be subjected to several mathematical procedures including multiplication and differentiation. But our purposes are fully satisfied if we merely know how to combine vectors and how to break them into directional components. The tail to tip method of combining velocity vectors and the shadow method of breaking velocity vectors into components will be presented.

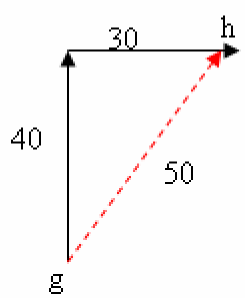
Demonstrating the addition of velocity vectors is easiest if we use the cardinal directions east, west, north, and south. By convention, the length of a velocity vector is drawn in proportion to the magnitude of the speed. The direction of the vector corresponds to the direction of motion.

Combining velocity vectors

The following scenario demonstrates how velocity vectors are added to each other. Suppose there are two forces acting on a planet. One makes it move 50 kilometers per hour north and the other 40 kilometers per hour east. The net velocity will be found by combining the two known components. We can combine them in any order as long as we obey the rule of connecting them tail to tip:



The net velocity is simply found by starting at the beginning and ending at the end.



$$\sqrt{40^2 + 30^2} = 50$$

The net velocity is, by pythagorus, 50 kilometers per hour in the direction of the red dashed vector.

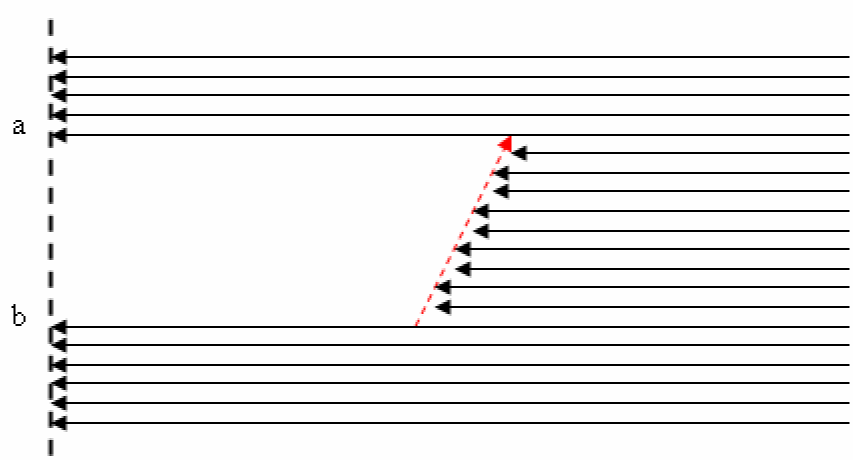
The net velocity was fairly easy to find , given all the information about two of its components that are at right angles to each other.

Note that the above scenario of combining velocity vectors requires no formal proof. The method is shown to be valid by applying a time interval to our velocity vector arrangements. For example, for the arrangement of vectors above, let two hours pass. The velocity vectors transform from representing velocity to representing a distance traveled. In two hours, traveling 80 kilometers north and then traveling 60 kilometers east would yield the same net result as starting at point g and traveling 100 kilometers directly to point h in the direction of the red dashed arrow.

Components of velocity vectors

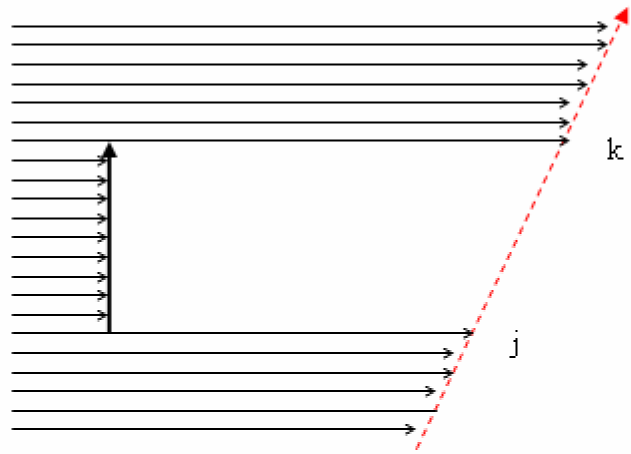
At times, calculating a component of velocity in a certain direction is desirable.

Suppose in the scenario above that the total velocity magnitude and direction was given and it was necessary to know how fast the planet was traveling in a northerly direction. A shadow method can be applied as follows. Draw a line in the direction for which we are interested in finding the velocity - in this case north. Let parallel rays of light shine perpendicular to the desired direction towards the known velocity vector. The shadow that the velocity vector casts on the direction line will be the magnitude of velocity along that direction line. The magnitude of velocity in a northerly direction is represented below by the distance from *a* to *b*. The shadow of the red total velocity vector on the north direction line gives the northerly component of total velocity.



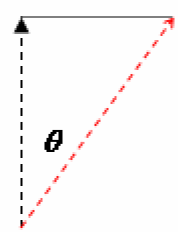
Finding the total velocity from a component

A known velocity component can be used to find the total velocity if the direction of total velocity is known. We simply cast the shadow of the component vector whose magnitude and direction are known on the direction line for true movement. The rays of light casting the shadow are oriented perpendicularly to the direction of the velocity component.



In the figure above, the northerly component of velocity of magnitude 40 kilometers per hour is known. Also known is the direction of movement of the planet represented by the orientation of the red dashed arrow. Perpendicular rays shine upon the known northerly velocity vector and cast a shadow upon the true direction line. The segment from j to k represents the magnitude of the total velocity. Knowing the angle between the northerly vector and the total velocity vector sets up a right triangle whose three angles are known. Using values for the cosine of the angle between the vectors it is possible to solve mathematically for the length of the total velocity vector if simply measuring the length of the red segment $\underline{kj}$ with a ruler is not accurate enough.

<u>Using the cosine of the angle between vectors</u>



Let the angle θ between the vectors be known. The hypotenuse of the right triangle is the total velocity vector. The cosine of θ is the north component vector divided by the total velocity vector:

$$\cos\theta = \frac{V_{north}}{V_{total}}$$

So the length of the total velocity vector is found by the equation :

$$V_{total} = \frac{V_{north}}{\cos\theta}$$

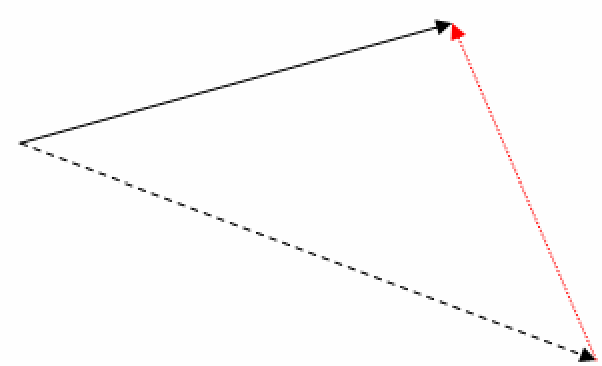
For the scenario above, the angle θ between the two vectors would be given to be 36.8 degrees. The known magnitude of the north component vector , 40 kilometers per hour, and the angle θ between the vectors give the total velocity as follows:

$$\cos\theta = 0.8$$

$V_{total} = \frac{40}{0.8} = 50$ kilometers per hour.

The shadow method and the cosine concept are thus shown to be useful in analyzing the relationship between total velocity and its components.

One final concept regarding vectors is that of representing changes in velocity. We have seen that we can add vectors tip to tip to result in a resultant or total vector. If we have a given initial velocity represented by a velocity vector and later in time we have a new velocity represented by a different vector, we can draw a third vector to represent the change in velocity. The change in velocity is simply represented by the tip to tip vector addition method. For example, in the diagram below let the initial velocity be represented by the dashed line. Then suppose an hour later the velocity has changed in magnitude and direction so that it is represented by a new vector represented in solid black. The change in velocity that would have occurred during the hour would be represented by the red arrow.



We will see that this concept of change in velocity is important in later chapters. We are all familiar with changes in velocity. A change in velocity during a given amount of time is defined to be an acceleration. Sometimes velocity changes without altering direction. For example, a car may accelerate along a straight road as it increases its speed. At other times the velocity changes in direction. For example, in the diagram above, the direction of the final velocity differs from the direction of the initial velocity. As this acceleration occurred, the magnitude of the velocity as well as the direction of velocity both changed.

Now we have a familiarity with vectors and the velocities which they represent. Vectors are essential to the understanding of orbits presented in *Orbits Explained*.