In Chapter 25 on areal velocity it was shown that for circular orbits $h = \sqrt{GMR}$. We also know from Chapter 24 that $T = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{GM}}$ for circles. We will apply these formulas to our knowledge that for a family of orbits in a solar system that have the same semimajor axis there is a family member that is a circular orbit whose radius is equal to the semimajor axis of all the elliptical orbits. We will gain the more general formula for elliptical orbits, $h = \sqrt{GMp}$.

Since within a family of elliptical orbits of equal semimajor axis there is a circular orbit of equal semimajor axis which is its radius, we have the advantage of knowing that their orbital periods are all equal since in Chapter 33 we showed that $T \propto a^{\frac{3}{2}}$ and we know that a can also represent the radius of the circular orbit in this family of orbits. Let's describe h, the areal velocity, for an ellipse and use the mathematical expression for the period of the circular orbit whose radius matches the semimajor axis of the ellipse since both have the same value.

$h = \frac{2\pi ab}{T}$ for an ellipse

Now substitute the formula above for period of circular orbits into the denominator of the formula for h:

$$h = \frac{2\pi ab\sqrt{GM}}{2\pi R^{\frac{3}{2}}}$$

Now within our family of ellipses of equal period and semimajor axes $a = R$, that is to say the semimajor axis length is the same as the radius of the circular orbital family member.

So we can substitute a for R in this context where period and semimajor axis are the same.

$h = \frac{2\pi ab\sqrt{GM}}{2\pi a^{\frac{3}{2}}} = \frac{b\sqrt{GM}}{\sqrt{a}}$ and since $p = \frac{b^2}{a}$ $h = \sqrt{GM}\sqrt{p} = \sqrt{GMp}$

And so we have the more general expression for areal velocity of the ellipse. So far we have used *a priori* methods for our derivations except for allowing the empirical finding of Gallileo that unequal masses fall at the same rate to the ground. This allowed us to determine that $\frac{2\pi}{\sqrt{GM}}$ is the constant that is used to turn the proportion for circular orbits $T \propto R^{\frac{3}{2}}$ into the equation $T = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{GM}}$. If we stick to more *a priori* methods we are limited to applying only proportions in this chapter and we would have been able to prove that $h \propto \sqrt{p}$ for elliptical orbits. Now this would be enough for us to proceed with but it will be easier in subsequent chapters that build the Energy Equation if we use the definitive equation instead of the proportion . But of note is that we could still proceed through all subsequent chapters using only the proportions and still be able to arrive at our final understanding of all topics. It is better to use the equations instead of only the proportions for another reason. In this way the equations will be familiar since they follow the conventions used in other texts.

Now let's continue in equation mode allowing for the empirical finding of Gallileo that unequal masses fall to the ground at the same rate.

We will transform the proportion , $T \propto a^{\frac{3}{2}}$, that we found in Chapter 33 by strictly *a priori* methods into the definitive equation, $T = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{GM}}$ allowing Gallileo's emprical finding.

We found in this chapter that $h = \sqrt{GM\sqrt{P}}$

Now we know $h = \frac{2\pi ab}{T}$ by definition of areal velocity for an ellipse.

Solving for T, and then substituting for h as above , and then substituting $\frac{\sqrt{a}}{b}$ for $\frac{1}{\sqrt{p}}$:

$$T = \frac{2\pi ab}{h} = \frac{2\pi ab}{\sqrt{GM\sqrt{p}}} = \frac{2\pi ab\sqrt{a}}{b\sqrt{GM}} = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{GM}}$$

And so we have in this chapter shown the general formulas for areal velocity and periods for elliptical orbits.