Areal Velocity

In this chapter we are interested in demonstrating a relationship between the rate of area swept by a planet and the gravitational constant.

As a planet orbits we know it sweeps out equal areas in equal times. This means that the area swept per unit time is constant. The expression for the rate of area swept is called the areal velocity. For geometrical reasons it is easier to speak of twice the area swept per unit time since that is what is actually swept when we multiply the radius by the arc that is swept. To obtain the true area swept we would have to divide this in half since our wedge is a triangle and not a rectangle. So by convention, in texts concerning celestial mechanics, areal velocity is h which equals twice the area swept in a unit of time.

Let's apply this to a circle to learn about areal velocity for circular orbits. Areal velocity will be slightly more complicated for elliptical orbits as we will see in a later chapter.

By the above definition, for a circular orbit:

$h = \frac{2\pi R^2}{T}$ which is by inspection twice the area of the circle divided by the period of the orbit.

We can see that this is a rate of area swept. We already know that this rate is constant within an orbit since equal areas are swept in equal times.

But let's compare different size circular orbits around the same central body.

In Chapter 24 we saw that $T = \sqrt{\frac{4\pi^2}{GM}} R^{\frac{3}{2}}$

So by substitution for T:

$$h = \frac{2\pi\sqrt{R}\sqrt{R}\sqrt{R}\sqrt{R}}{2\pi\sqrt{R}\sqrt{R}\sqrt{R}}\sqrt{GM} = \sqrt{R}\sqrt{GM}$$

Now, for a solar system GM is a constant since the mass of the Sun does not change and neither does the gravitational constant.

So if we picture planets in circular orbit around the same Sun at various distances, we know that the rate of sweep of area for all these planets is proportional to the square root of the radius.

We can think of GM as being the amount of gravity at the Sun since the expression implies a constant related to gravitational force, G, multiplied by the mass, M, of the Sun.

So we can say that the rate of area swept for circular orbits in a solar system is proportional to the square root of the radius and that the amount of gravity at the Sun is what regulates the proportion.

In a later chapter we will see in *a priori* fashion that for an ellipse the more general expression for areal velocity is:

$$h = \sqrt{GM}\sqrt{p}$$

where p is the semilatus rectum of the ellipse