

Third Law Circular Orbits

In this chapter we will show that for circular orbits the period of an orbit is proportional to $R^{\frac{3}{2}}$, the square root of the cube of the radius. In Chapter 33, we will apply this proportion to logic that will prove that the period of an elliptical orbit is proportional to the square root of the cube of the semimajor axis.

We will actually find more than just a proportion in this chapter. We will find an equation relating the radius of the circular orbit to the period of the orbit.

In Chapter 21 we saw that $F = \frac{GMm}{R^2}$.

We saw in Chapter 22 that for a circular orbit,

$a = \frac{v^2}{R}$ and that $F = ma$.

So we can say $F = ma = m \frac{v^2}{R} = \frac{GMm}{R^2}$ so canceling out

m :

$$\frac{v^2}{R} = \frac{GM}{R^2} \quad \text{so} \quad GM = \frac{v^2 R^2}{R} = v^2 R$$

We showed that for a circular orbit $v = \frac{2\pi R}{T}$

So substituting for v we get $GM = \frac{4\pi^2 R^2 R}{T^2}$

Solving for T^2 we get:

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2}{GM}} R^{\frac{3}{2}}$$

So we have shown that for circular orbits the period is proportional to the square root of the cube of the radius.

Note that we used some empirical information in Chapter 22 to get the coefficient GMm for the equation relating force to distance. But we knew in *a priori* fashion that force is inversely related to the square of the distance. If we used only the *a priori* information, that is to say if we only used the proportion and not the full equation, our derivation above would have yielded the result:

$T \propto \sqrt{4\pi^2 R^2}^{\frac{3}{2}}$ which states that $T \propto R^{\frac{3}{2}}$ which is all we really require for later *a priori* proofs. In other words, our transgression into empirical methods will not contaminate our *a priori* scheme. The transgression does nicely serve to deliver the relationship between the period and the radius in terms of an equation instead of in terms of a proportion and therefore is more comfortable to work with since we are all familiar with calculating with equations.