Circular Acceleration

In this chapter we will see that for a planet in a circular orbit, the acceleration is equal to

$$\frac{v^2}{R}.$$

Recall that in Chapter 4 we described how to use vectors to represent changes in velocity.

Acceleration is defined to be the change in velocity in a unit of time so:

$$a = \frac{\Delta v}{\Delta t}.$$

Velocity is the distance traveled in a unit of time. The distance of a circular orbit is $2\pi R$ so the velocity is:

$$v = \frac{2\pi R}{T}.$$

Rearranging:

$$T = \frac{2\pi R}{v}$$

In a circular orbit the velocity is constant. It changes in direction by 360 degrees during the course of one orbit. Recall that the hodograph for a circular orbit has a radius equal to the velocity of the planet since the hodograph is assembled from the arrows that represent the velocity of the planet as it circles the Sun. The origins of the arrows are placed at a single point and the tips land on the circle of the hodograph as explained in a previous chapter. The change in velocity is the length of the arc of the circle so:

$$\Delta v = \frac{2\pi v}{T}$$

Combining our equations for distance and velocity:

$$\frac{\Delta v}{\Delta t} = \frac{2\pi v}{\frac{2\pi R}{v}} = \frac{v^2}{R}$$

So we see that for a circular orbit $a = \frac{v^2}{R}$.

We will use this equation to prove Kepler's Third Law.