The Gravitational Constant

It is time to make an exception to *a priori* methods. We have found in *a priori* fashion that gravitational force is inversely proportional to the square of the distance to the Sun. In Chapter 21 when we conclude that $F \propto \frac{1}{R^2}$ we are saying that $F = k\frac{1}{R^2}$ where k is a constant. The problem is that *a priori* methods will not suffice to quantify the constant k . No, for that we will need to use empirical methods to define what the constant stands for. All is not lost since we will not need this empirical derivation for our proofs. Our proofs will still qualify as being *a priori* since they require only the proportion between force and the inverse square of distance and do not require the refinement of the constant k. But it will be useful to define our constant so that the proportion will take on a more recognizable form- that of a finished equation. In this chapter we will show that the finished equation is :

$F = \frac{GMm}{R^2}$. (For our purposes in this chapter the smaller mass m is a falling object and the larger mass M is that of the central attracting body in this case the mass of the earth. In general, the smaller mass will be the planet and the larger mass the Sun for later chapters.)

Our empirical digression will be only to accept the findings, of Galileo, that objects fall to the ground at equal rates regardless of their mass. Now this is hardly an astronomical observation and could almost be classified as being part of everyday intuitive earthbound experience - and yet it is difficult to accept as *a priori*. But if we accept the fact that objects fall at the same rate we can use logic to get from our proportion to the finished equation.

There is a nice discussion of this chapter's intended message in Azimov's book on Physics. I will try to restate it. Let's accept that a large mass dropped from a given height such as a rooftop causes more damage than a small mass. This is the clue that tells that mass is related directly somehow to force. As force increases, so does damage. So mass belongs in the numerator of our equation.

Let's also propose that for our gravitational force equation, as for any equation that is born out of a proportion, an arbitrary constant can be assigned to participate in the regulation of the proportion. This will be our constant G in the above equation, the gravitational constant.

Now the only question as to how to set up an equation involving force, distance, mass, and the gravitational constant is; How do we treat the masses. For example, the equation could deal with the sum of the masses or their product or some other arrangement. But based on the findings of Galileo there can be only one arrangement. The only algebraically possible arrangement is to use the product of the masses. Only in this manner will the effect of the falling mass cancel out, allowing differing masses to fall at the same rate. The smaller mass cancels out as follows:

$F = ma$ so the acceleration and rate of fall of a mass m are determined by the force.

Now if the product of the masses is the correct arrangement for the equation, or in other words:

If $F = \frac{GMm}{R^2}$, the acceleration separates nicely from the mass and is $\frac{GM}{R^2}$, thus the small mass m does not participate in determining what the acceleration is going to be. In other words the small mass m does not influence the rate of fall.

Note that algebraically we could not ignore the small mass in the same way if the equation for force depended upon the sum of the masses instead of the product of the masses:

If $F = G\frac{M+m}{R^2}$ we could not separate out m neatly from F .

The mathematical meaning of not being able to separate F and m by a single expression that can be contained within parentheses is that mass influences force. And if that were the case, varying masses would fall to earth at varying rates.

So our equation becomes $F = k^{'}\frac{Mm}{R^2}$ since we find that the product of the masses is the correct arrangement. We are arbitrarily allowed to rename $k^{'}$ to be G and to call it the gravitational constant.

G represents the rest of the constant of proportionality (already having accounted for masses) between force and the inverse square of distance. We need not further characterize what G is for our later proofs - it is there so that we have an equation to use later on, especially when we derive the Energy Equation. But the nature of curiosity invites us to theorize by inspection of the equation, that G is the constant that relates how much gravitational force is generated per unit of mass.