The Inverse Square Law

Let it not be understated that this chapter is one of the milestones toward which we have been headed. It is the point at which we can say that Kepler's First Law, which was proven in Chapter 11, and now the Inverse Square Law, have both been shown to stand on their own. In one sense this chapter is The End. The main objective is met. But as you can see this chapter is somewhere in the middle of the book so there will be more to follow. Specifically, in later chapters I will pursue the *a priori* proof of Kepler's Third Law concerning the periods of planets, a proof with novel methods close to folly; astounding for being at the same time seemingly contrived, yet undeniably true. And in still later chapters the Energy Equation will be derived.

I have used no empirical astronomical data. In this chapter I will show, in *a priori* fashion, that the centrally directed force is inversely proportional to the square of the distance. In other words the gravitational force of the Sun is inversely proportional to the square of the distance to the planet.

The change in velocity with the change in angle reveals the force law

We know from basic physics that force is defined as the change in velocity per unit time. The change in velocity per unit time is defined as acceleration. The basic formula for force is $F = ma$ where m is the mass of the accelerating object and a is the acceleration of the object.

The change in velocity per unit time can be written as $\frac{\Delta v}{\Delta t}$.

So $F = ma = m\frac{\Delta v}{\Delta t}$. Notice how this fits with the definition of force as the change in velocity per unit time.

We are interested in proportions. Proportions, logic and algebra will provide the proof.

So $F \propto \frac{\Delta v}{\Delta t}$

Now algebra tells us we can write:

$$\frac{\Delta v}{\Delta t}=\frac{\Delta v}{\Delta \theta}\times\frac{\Delta \theta}{\Delta t}$$

Now $\frac{\Delta \theta}{\Delta t}=\omega$ which we recognize as angular velocity from the previous chapter.

We also found that $\omega \propto \frac{1}{R^2}$

So by combining the equations above: $F \propto \frac{\Delta v}{\Delta \theta}\times\frac{1}{R^2}$

Note that (as shown by Feynman and Goodstein and Goodstein, and as demonstrated *a priori* in Chapter 20) for an orbit, $\frac{\Delta v}{\Delta \theta}$ is constant. And so, in *a priori* fashion we can conclude:

Thus $F \propto \frac{1}{R^2}$.