

Change in Velocity is Proportional to Change in Angle

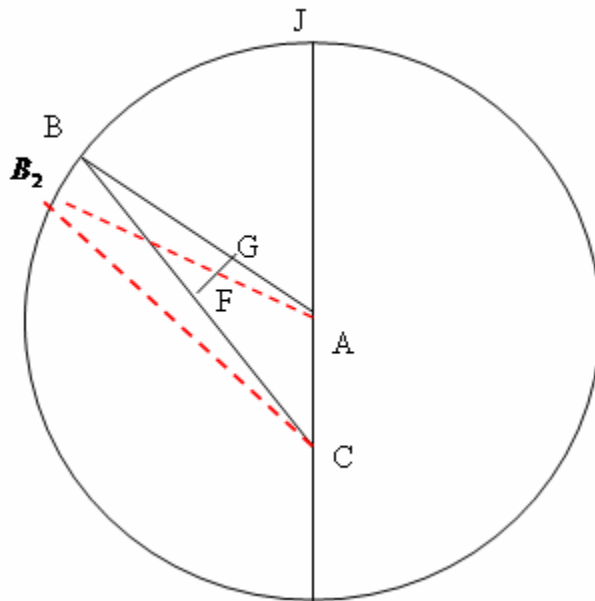
There is a proportion that we will use in subsequent chapters that relates to the velocity changes that occur as the planet moves in relation to the Sun. In this regard, this chapter presents a brilliant insight into the hodograph that is credited to Richard Feynman in "Feynman's Lost Lecture" by David Goodstein and Judith Goodstein.

Specifically, Feynman's observation that $\frac{\Delta V}{\Delta \theta}$ is a constant will be demonstrated.

Recall that in Chapter 4 we described how to represent a change in velocity using velocity vectors.

Note that the Inverse Proportion Machine spins to create a circular velocity diagram. The long segment \overline{AB} spins around the tip of the short segment \overline{AC} so that the point B traces a circle. The total velocity vectors have been shown to be the segment \overline{CB} in the hodograph. The angle from perihelion position has been shown to be the angle $\angle JAB$. The total velocity vector \overline{CB} always ends on the circle meeting the unique radius \overline{AB} that indicates the angle from perihelion for the planet. The total velocity

vector \overline{CB} changes position and length continuously as the planet moves continuously along the ellipse. The change in total velocity from one moment to the next is represented by the small arc $\overline{BB_2}$ along the velocity circle that is cut by the small change in the angle from perihelion as the planet moves. In other words the angle $\angle JAB$ changes and induces a proportional change in the length of the arc of the circle that the angle spans. That small arc $\overline{BB_2}$ of the circle is the change in velocity since it is the vector that must be added to the first total velocity vector \overline{CB} in order for the result to be the second velocity vector $\overline{CB_2}$.



Since the arc $\overline{BB_2}$ of the circle represents the change in velocity and since the change in the angle $\angle JAB$ to perihelion is proportional to arc $\overline{BB_2}$, we can say that the change in the angle to perihelion is proportional to the change in the velocity. Mathematically this means that $\frac{\Delta V}{\Delta \theta}$ is a constant where V is the total velocity and θ is the angle to perihelion. This relationship will be instrumental in the *a priori* proof that gravitational force varies inversely with the square of the distance between the masses.