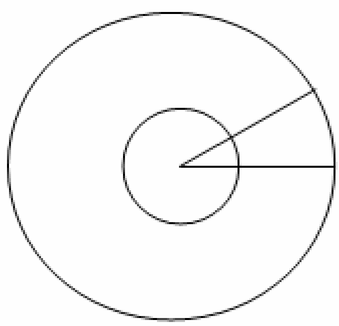
Angular Velocity to Radius

We are back on track toward the stepwise *a priori* proofs concerning orbits. As with many subsequent chapters, a small portion is presented here. In this chapter we show that within an orbit the angular velocity is inversely proportional to the square of the distance to the Sun. We call the angular velocity ω.

The angle swept per unit time

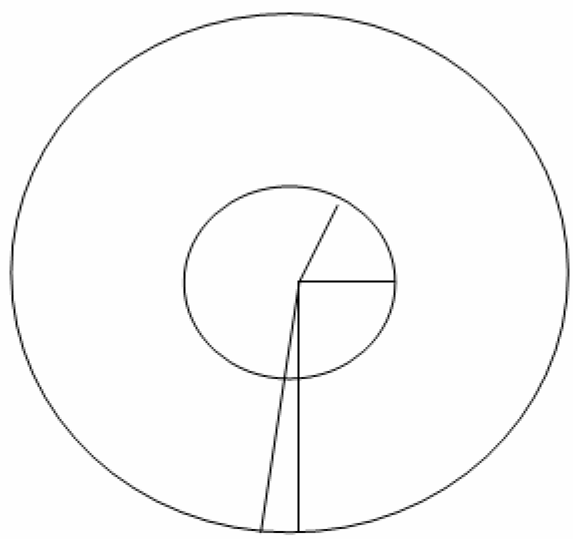
For any orbit we know that equal areas are swept in equal times. It is valid to look at very small time intervals and examine the thin wedge of area that is swept. Each thin wedge approximates the wedge that would be swept for a circular orbit. Only the very short arc at the far end of the wedge varies slightly in contour from the arc traveled by the circular orbit. As the time interval gets shorter, this arc is essentially equivalent to the arc traveled in the circular path.

Compare the area of wedges that span the same angle relative to the Sun but differ in radius distance from the Sun. The area of a circle is proportional to the square of the radius of the circle.



It follows that when we cut these circles into wedges of equal angle, the area of the wedges of equal angle will also be proportional to the square of the radius.

Now hold the wedge area fixed and let the angle of the wedge vary. The radius must change if the area is to remain constant. How exactly does it vary?



For a given circle, the area of a wedge is proportional to the angle of the wedge. We showed above that the area of a wedge grows in proportion to the radius squared.   So to keep the area constant as the radius changes the angle must decrease in proportion to the radius squared. For example, it the radius of a wedge doubles, the angle of the wedge must be quartered in order to keep the area of the wedge constant.

So to keep areas constant, the angle swept for each wedge swept must decrease in proportion to the square of the radius of the wedge. Now recall that equal areas are swept in equal times. So area is equivalent to time. So substitute time for area in the first sentence. To keep times constant, the angle swept for each wedge must decrease in proportion to the square of the radius of the wedge. In other words we have found that the change in angle per unit time is inversely proportional to the square of the radius.

Restated, the angular velocity is inversely proportional to the square of the radius.

$$\omega \propto \frac{1}{R^2}$$