

Let's pause for some philosophy. We note that we see that the segment \overline{HC} is the component of total velocity that is in the direction of the line connecting the planet to the Sun. We call this component of velocity the radial velocity since it is in line with the radius pointing from the planet to the Sun. In this book we do not pay much attention to the radial velocity but it is there for anyone to observe and ponder. By drawing several representative hodographs it becomes obvious that the segment \overline{HC} is shared by Inverse Proportion Machines that are oriented like the one above with the long segments \overline{AB} on a straight line at 180 degrees from each other. We conclude that for every elliptical orbit, the radial velocity of a planet at an angle θ from the Sun is equal but in opposite direction to the radial velocity of that planet when it is at $\theta+180$ degrees relative to the Sun. Note figure a:

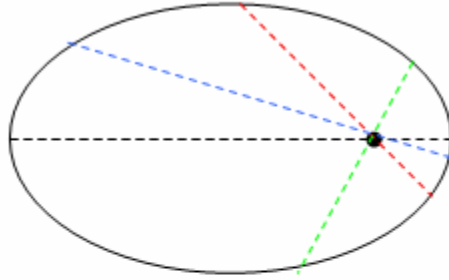


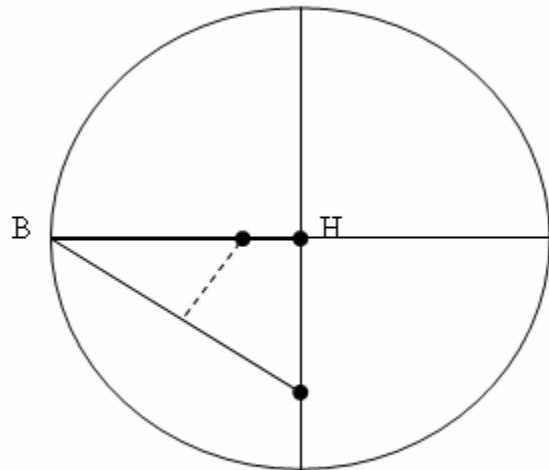
Figure α : Let the sun be at the focus represented by the black point. Each dashed line can be designated to be a "Diameter of the Ellipse Relative to the Sun at the Prime Focus."

At any combination of these "Diameters of the Ellipse Relative to the Sun at the Prime Focus" the planet falls toward the central body as much as it escapes from the central body. And so that is why it stays in orbit.

There is another observation to make regarding these "Diameters of the Ellipse Relative to the Sun at the Prime Focus." This one concerns tangential velocity.

Notice that the tangential velocity for any angle added to the tangential velocity for 180 degrees away sums to a constant. We can see that in the hodograph above with the two Inverse Proportion Machines . Examine the segment \overline{BH} in both Inverse Proportion Machines. The two

\overline{BH} segments sum to a constant. This applies to any combination of positions for the planet that are 180 degrees away from each other as measured relative to the Sun. The sum is equal to exactly twice the tangential velocity when the planet's radius is at 90 degrees from perihelion. This occurs when the planet is at the tip of the "Sunny side" semilatus rectum:



The segment \overline{BH} , representing tangential velocity is equal to the half the diameter of the velocity diagram of the hodograph. Its magnitude represents half the sum of the tangential velocities for pairs of positions at 180 degrees from each other since we saw above that the sum is equal to the diameter of the velocity circle. It is thus

equal to half the sum of the tangential velocity at perihelion and aphelion. Since tangential velocity is equal to total velocity at those positions we can say that the tangential velocity at the semilatus rectum position is equal to half the sum of the total velocities at aphelion and perihelion positions. In simpler terms, consider the speed of the planet when it is closest to the Sun and when it is farthest from the Sun. Take the average of these two speeds and the result is the tangential velocity of the planet when it is at the end of the "Sunny side" semilatus rectum of its elliptical orbit. We are merely being philosophical here. Although these relationships are interesting, we will not use them directly in our proofs. However, it is noteworthy that there is a historical parallel here. Recall the notion of the equant offered by Claudius Ptolemy in the second century A.D.. The equant was a place within a circular orbit where the angle to a planet changed at a constant rate. Here we have a relationship between tangential velocity, not exactly angular velocity as addressed by Ptolemy, and the Sun. Like Ptolemy, we appreciate these kinds of patterns and search for them along the way in our studies.