Right Angle Velocity in Correct Proportion

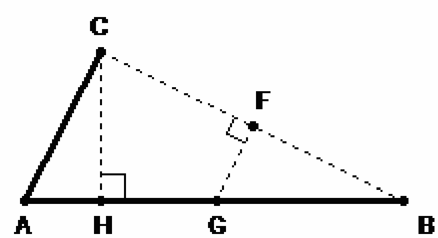
In this chapter the Inverse Proportion Machine is studied to see how it represents the planet's radius to the Sun and tangential velocity. We have a new tool and want to learn how to apply it.

When a batter strikes the baseball and watches as the ball travels high and deep toward the home run wall, he or she is not so concerned about how high the ball has been struck. Rather, it is more important how far the ball flies from its origin at home plate. In effect, we ignore the vertical aspect of the trajectory of the ball and concentrate on the distance as measured from home plate to the outfield stands in fair territory.

Similarly, we are concerned in this chapter with only one aspect of the velocity of a planet. If we draw a straight line from the planet to the Sun, we are interested in the speed of the planet at a right angle to that straight line. That speed is conventionally called tangential velocity and can be accurately represented by a velocity vector.

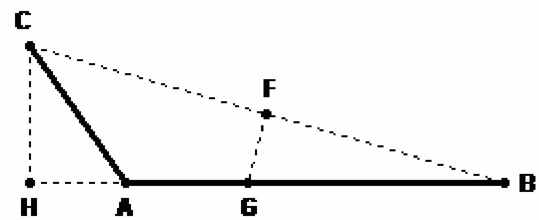
Tangential velocity oriented correctly

We can show that the segment $\overline{HB}$ is a valid representation of tangential velocity. Let's review the Inverse Proportion Machine spinning to create the hodograph.

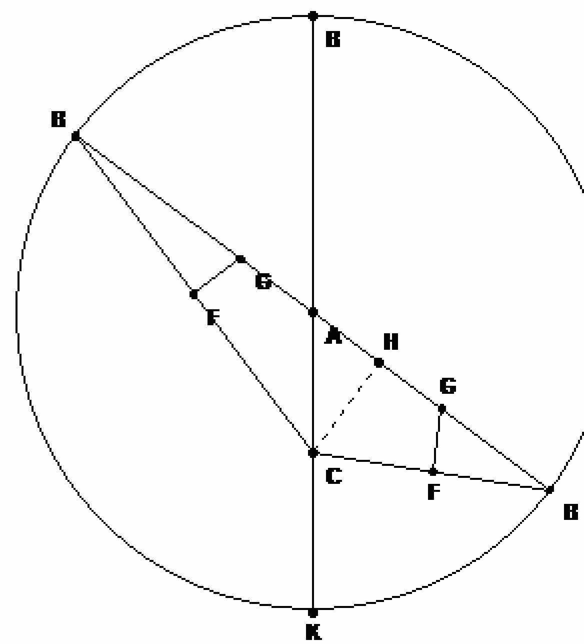


In the figure the segment $\overline{AB}$ is allowed to spin at point *A* so that it spins around the shorter segment $\overline{AC}$. As this happens we know from the derivation of the properties of the Inverse Proportion Machine in a previous chapter that the distance $\overline{HB}$ is inversely proportional to the distance $\overline{AG}$.

We also recall that same proportion holds true for the other position of the Inverse Proportion Machine wherein the angle between the segments is greater than 90 degrees:



Two positions of the Inverse Proportion Machine are shown in the hodograph below:



In Chapter 4 we saw that for an orbiting planet, the velocity that is at a right angle to the line to the Sun is called the tangential velocity. We showed in Chapters 5 and 6 via geometry and the concept of sweeping equal areas in equal times, that the tangential velocity must be inversely proportional to the distance to the Sun. In Chapter 11 we explained the preference for choosing point G to represent the position of the planet and the stationary point A to represent the position of the Sun. In that case the radius from the planet to the Sun , $\overline{AG}$, is inversely proportional to the tangential velocity, $\overline{BH}$.

The 90 degree convention for the ellipse

Recall that in Chapter 7 we showed that for a circular orbit that the tangential velocity on a hodograph is at 90 degrees to the true tangential velocity of the planet. To make the same point in a different way, notice that we are assigning the properties of radius and tangential velocity to segments on the hodograph. Those segments lie on the same straight line. But they truly can not lie on the same line. Since by definition, tangential velocity is always at a right angle to the line to the Sun from the planet, we have our “instructions” to rotate the velocity diagram 90 degrees relative to the line containing the radius to the planet on the hodograph.

The 90 degree rotation is relative. In other words when rotate the tangential velocity by 90 degrees we are doing it relative to the orientation of the radius. We would achieve exactly the same relative orientation, that is a 90 degree relationship between radius and tangential velocity, if we rotate the radius on our mind by 90 degrees instead of rotating the tangential velocity. In that case our convention would be that the true radius and true position of the planet is always 90 degrees away from where it is represented on its hodograph. In some cases rotating the radius by 90 degrees is easier to visualize than trying to visualize a 90 degree rotation of the velocity. So if the hodograph is drawn so that the velocity vectors are pointing in the true correct direction , it generates an ellipse that represents the orbit of a planet which must be rotated by 90 degrees in order to represent the planet and its orbit in correct orientation. On the other hand if we have a hodograph that represents the correct orientation of the elliptical orbit, we must rotate the velocity vectors 90 degrees in order to obtain the true direction for velocity.

It is best to restate our 90 degree rotation convention as follows. When interpreting the hodograph, the correct spatial orientation will result from rotating the major axis of the ellipse by 90 degrees if the velocity directions on the hodograph are to be taken literally. Equally valid is to take the orientation of the major axis of the ellipse to be correct and rotate the direction for all velocity vectors by 90 degrees.

In summary, the tangential velocity is represented by the segment $\overline{BH}$ in the hodograph. It is important to remember to orient position and velocity correctly relative to each other.