# Orbits are Ellipses

This chapter brings together the themes of chapters 5, 6, and 9; the concept of area swept, the inverse relationship between tangential velocity and radius, and the hododyne. These themes are combined in order to prove in *a priori* fashion that orbits are ellipses.

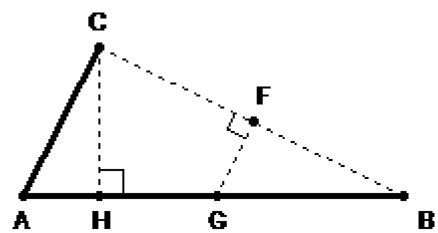
Johannes Kepler's realization that the ellipse is the shape of planetary orbits came to him only after a long period of frustrating and seemingly fruitless mathematical analysis. When the revelation came, he knew that it was a monumental discovery. In fact his planetary laws made him one of the most widely known and recognizable luminaries in Europe. However, this did not make his life comfortable; despite fame and intellectual satisfaction, he continually struggled against financial difficulties and religious opposition. Those who appreciate Kepler may be comforted by the fact that his work lives on and that many of his original manuscripts survive. This chapter presents the first *a priori* proof of Kepler's First Law - that planets travel in elliptical orbits.

We know by our definition of an ellipse that the sum of the lengths of the legs of an ellipse is a constant. The scheme here is to show that the obligatory path of the planet, determined by the hododyne, satisfies this definition.

## Spin the Inverse Proportion Machine to form a circle

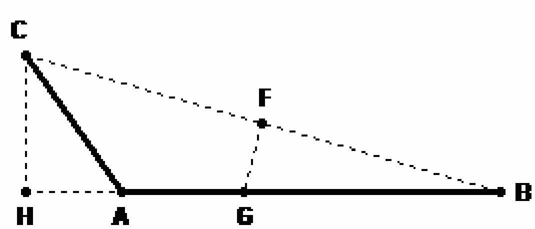
The string and tack construction of an ellipse as described in a previous chapter is the method by which the Inverse Proportion Machine, a.k.a. the hododyne, generates valid planetary orbits. In this chapter we will see that as the Inverse Proportion Machine spins, the legs of an ellipse are generated in a manner consistent with our mathematical requirements.

We saw in chapter 9 that the Inverse Proportion Machine begins as a straight line of finite length that is bent into two unequal segments. The long segment is allowed to spin, at the bend, around the tip of the short segment. In the figure below is an example of an Inverse Proportion Machine for us to analyze.

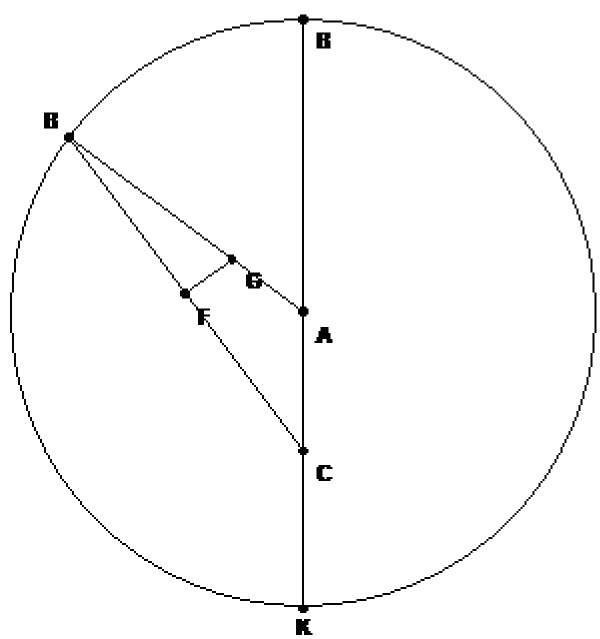



In the figure the segment $\overline{AB}$ is allowed to spin at point *A* so that it spins around the shorter segment $\overline{AC}$. As this happens we know from the derivation of the properties of the Inverse Proportion Machine in Chapter 9 that the distance $\overline{HB}$ is inversely proportional to the distance $\overline{AG}$.

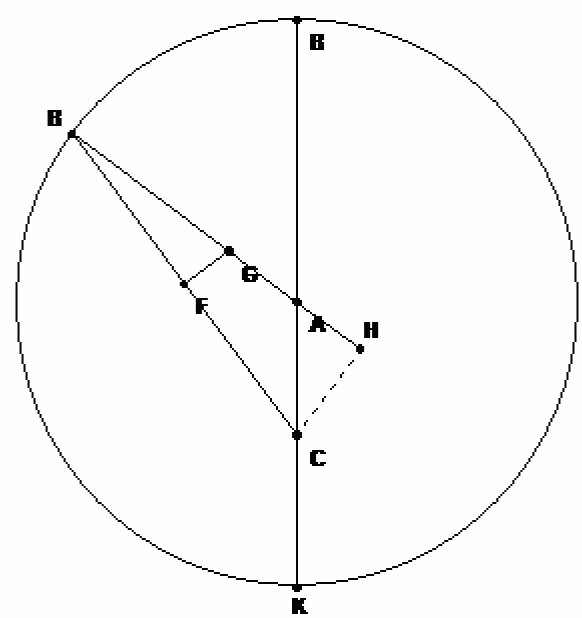
We also recall that same proportion holds true for the other position of the Inverse Proportion Machine wherein the angle between the segments is greater than 90 degrees:



We can see that as the long segment spins around point *A*, the point *B* will trace out a circle with a radius equal to the length of segment $\overline{AB}$. Let's visualize what the circle will look like. We see the resultant circle of radius $\overline{AB}$. We choose to display an arbitrary position of the Inverse Proportion Machine within the circle that it generates. We will show first only the segments $\overline{AB}$, $\overline{AC}$ and $\overline{FG}$:

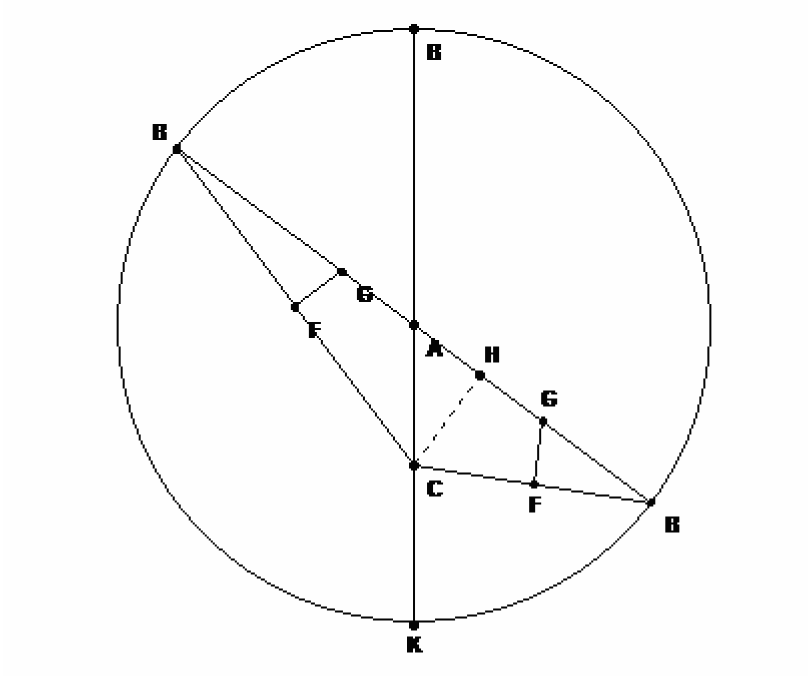



Next we can draw the rest of the Inverse Proportion Machine at this position to display the point *H*.

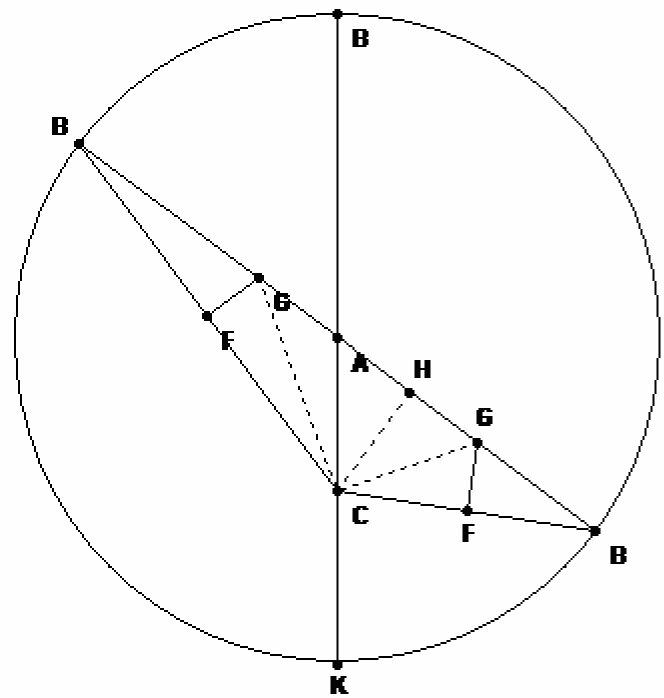



<u>The Sun is in the diagram</u>

So far, note that we have an entire Inverse Proportion Machine situated in the circle. We know that the machine dictates that as the Inverse Proportion Machine spins to generate the circle, the segment $\overline{AG}$ is inversely proportional to the segment $\overline{BH}$. The segment $\overline{AB}$ will spin to generate the circle. The center of the circle is thus at point $A$. Since point $A$ is stationary as the machine spins we can assign point $A$ to be the position of the central body, the Sun




We add lines to connect points *C* to points *G*:




We can see that $\overline{CG} = \overline{BG}$ within each individual position of the machine since point $F$ bisects segment $\overline{CB}$ for any position of the machine thereby creating two equal triangles *BFG* and *CFG*. Now since $\overline{AB} = \overline{AG} + \overline{GB}$ and since $\overline{AB}$ is a constant as the radius of our circle, then by sustitution, since $\overline{CG} = \overline{BG}$ then $\overline{AG} + \overline{CG}$ is also constant. This means that the sum of the lengths $\overline{AG} + \overline{CG}$ are constant everywhere around the circle and the lengths $\overline{AG} + \overline{CG}$ behave like the tack and string legs of an ellipse causing the point $G$ to follow strictly an elliptical path as the inverse proportion machine spins.

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## The planet is in the diagram

We can assign the distance $\overline{AG}$ to be the radius to the planet so that point $G$ will be the position of the planet as the machine spins. Since we know that the lengths of the segments are inversely proportional , it is valid to assign the tangential velocity the length $\overline{HB}$ and to assign the radius, or distance to the Sun, the length $\overline{AG}$. We chose $G$ of the segment $\overline{AG}$ to be the planet and its distance to the Sun respectively, instead of the segment $\overline{HB}$ and one of its points because the legs of the ellipse are latently represented in the Inverse Proportion Machine and meet at the point $G$ thus rendering the segment $\overline{AG}$ the natural choice for the radius and $G$ the natural point for the planet..

We see in *a priori* fashion then, that as the Inverse Proportion Machine spins, its segment $\overline{AG}$ causes point $G$, representing the position of the planet, to trace an elliptical path.